Solid Waste Facility Location and Transportation Model
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Engineering 66

Abstract
An AMPL mixed integer model is devised to assess the best transportation methods for municipal solid waste (MSW) in the form of recyclable materials. 20 collection points throughout a district must deliver 1000 tons of recyclable material to 2 processing plants, with the possibility of using 5 transfer facilities, which can be used to make transportation more efficient with the use of larger trucks.
Introduction

While the rate at which Americans generating waste on a daily basis has remained at about 4.62lbs for over a decade, it is still an undeniable fact that the waste must be disposed of in some fashion, whether it be to a landfill, for composting, or for recycling. The mere disposal of waste generates much concern for the environment, and factors including pollutants emitted from incineration facilities, emissions from vehicles transporting the waste and the recovery of recyclable materials creates the need for multiobjective models that can efficiently allocate the best resources for each task.

Theory

The main idea behind this exercise is the transshipment model which is used in order to solve the resource allocation problem of getting waste or recyclable material from the city center to the plants that can process them. The 20 centers are shown on the left and the 2 plants to which their material must arrive is shown on the right.

Figure 1. A transshipment model shown in a network flow diagram form. Note that one component is missing – the exercise allows for the direct shipment of material from the centers to the plants, which would bypass the middle column of transfer facilities.
To solve this problem we start with the initial model of trying to minimize total cost. The costs associated were:

a) transportation costs moving the waste from center to plant, center to transfer facility, if necessary, and from transfer facility to plant.

\[ \sum_k \sum_j a_{kj} z_{kj} \] where \( a_{kj} \) is the transportation cost, per ton of waste, and \( z_{kj} \) is the actual waste amount (the decision variable).

\( a_{kj} \) can then be calculated from

\[ a_{kj} = t_i d_{kj} \] where \( t_i \) is the cost per km-ton of transporting waste from center to transfer facility, and \( d_{kj} \) is the distance from center \( k \) to plant \( j \).

b) fixed costs associated with building a transfer facility

c) processing costs per ton of waste received at a transfer facility

\[ p x_{ki} \] where \( p \) is the cost/ton of waste and \( x_{ki} \) is the amount of waste going from center \( k \) to transfer facility \( i \).

The constraints are critical to solving the problem:
The associated constraints are:

a) The processing limit on the transfer facilities:

\[ \sum_k x_{ki} \leq y_i Q, \forall \text{transfers} \] where \( x_{ki} \) is the amount of waste going into transfer facility \( i \) from centers \( k \) which must be less than the capacity of a plant, \( Q \), if it is built, \( y_i \).

b) The plant capacities

\[ \sum_k z_{kj} + \sum_i w_{ij} \leq G_j, \forall \text{plants} \]

\( z_{kj} \) is the amount arriving from the centers, and \( w_{ij} \) is the amount arriving from the transfers. \( G_j \) is the capacity for plant \( j \).

c) The mass balance (what goes in must come out)

\[ \sum_k x_{ki} - \sum_j w_{ij} = 0, \forall \text{transfers} \] where \( x_{ki} \) is the amount arriving at transfer facility \( i \) and \( w_{ij} \) is the amount being sent out to plant \( j \).
d) All of the recyclable materials deposited at the centers must be picked up:

\[ \sum_i x_{ki} + \sum_j z_{kj} = S, \forall \text{centers} \]

A simple version of these equations are elaborated in the text, ‘Solid Waste Management’ by Jon C. Liebman, in “Design and Operation of Civil and Environmental Engineering Systems” by ReVelle and McGarity.

**Procedure**

For the first part of the exercise, an AMPL model was formulated with the goal of minimizing cost (see theory). Furthermore, a binary decision variable is added in order to allow for the possibility of not constructing a particular transfer station, if there isn’t the need for that facility.

In the second part of the exercise, another objective is assessed in order to reduce the environmental impact of the transportation of waste by minimizing the use of the transfer facilities 1, 2 and 3 because they are located near residential neighborhoods. This is important because larger trucks are used to transport waste in bulk from transfer facilities, and noise and environmental impact is a more important factor.

There are two approaches that can be made in the multiobjective analysis (as described in Cohon, Rothley’s chapter ‘Multiobjective Methods’ in Revelle and McGarity’s book, “Design and Operation of Civil and Environmental Engineering Systems”).

**Weighting method:** The weighting method divides a multiobjective problem into multiple single objective problems each with a weight \( w \) which can be either predetermined or set as another variable. Doing so ascribes a relative importance of one objective to another, thus allowing decision makers to assess the cost and benefit of their decisions. If the weights are derived analytically, decision makers can assess whether or not each individual objective’s needs and costs are worth the weight assignment they are given. This approach may be inefficient, however.

**Constraint method:** The approach taken in this exercise is the constraint method, taking advantage of the fact that the second objective of reducing environmental impact can be rewritten in the form of reducing the amount of waste that uses those resources that impact the environment negatively. Hence, all that is needed is a constraint that limits
the amount of waste allowed through facilities 1, 2 and 3, and then to vary that constraint incrementally to observe its effect on total cost.

**Results**

The minimum cost associated with transporting waste from the centers to the plants was obtained when

<table>
<thead>
<tr>
<th>Transfer facility</th>
<th>Tons transported</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant</td>
<td></td>
</tr>
<tr>
<td>1 → 1</td>
<td>2000</td>
</tr>
<tr>
<td>1 → 2</td>
<td>2000</td>
</tr>
<tr>
<td>2 → 2</td>
<td>4000</td>
</tr>
<tr>
<td>3 → 2</td>
<td>4000</td>
</tr>
</tbody>
</table>

From this layout it can be easily seen that plant 2 takes all of its input from transfer facilities, which is to be expected given that the distances from transfer facilities to plant 2 is in general larger than those from transfer facility 1. That is, it wouldn’t be economical to use transfer facilities and to pay for processing at those facilities if those costs could not be recovered by the reduced costs afforded by transporting waste from transfer facilities in more efficient vehicles (it costs half as much per km to transport waste from a transfer facility as it does to transport waste from a center to a plant or from a center to a transfer facility).
**Making the model more realistic**
In order to assess the sensitivity of the model as well as to consider features that would enable it to be a more realistic reflection of the real world, the following things were done:

**Table 1. Assessing the sensitivity of cost parameters to the model**

<table>
<thead>
<tr>
<th>Modification</th>
<th>Effect/Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increased transportation cost/km from center to plants to 2.5/km</td>
<td>Objective minimal cost increased to 478500, transfer facility 5 now in use</td>
</tr>
<tr>
<td>Reduced transportation cost from centers to transfer facilities to 1.5/km</td>
<td>Objective minimal costs decreased to 441000, transfer facility 5 also now in use</td>
</tr>
<tr>
<td>Increased transportation cost from transfer facilities to plants to 2.5/km</td>
<td>Objective minimal costs increase to 560000, all waste is now sent directly from center to plant.</td>
</tr>
</tbody>
</table>

**Multiobjective Analysis**
As described in the theory, the multiple objective programming approach taken here is constraint based, such that the amount of waste allowed to travel through transfer facilities 1, 2 and 3 are reduced below optima (which occurred at 12,000 units, for an optimum cost of $455,000). The graphic below attempts to describe how the constraint limiting affects the decisions made by the AMPL CPLEX solver, whereby transfer facilities 1, 2, and 3 are incrementally used less.
Figure 2. Cost versus varying environmental impact through transfer stations. Environmental impact is here defined only as a function of limiting the number of waste that can pass through key transfer facilities within the residential neighborhood. For further details see text.

From the figure above it can be seen that as the ‘environmental impact’ is reduced (going leftward), the transfer facilities 1, 2, and 3 shown in the boxes below are used less and less.

**Weighting approach**

An attempt is made to see the effect of adding a weighting factor between the cost and the environmental impact caused by using the transfer facilities located in town, without specifically attempting to normalize the ‘costs’ associated with environmental impact. This is potentially problematic, because in a sense it is ascribing one dollar’s worth of impact for an increase in one unit of waste traveling through the transfer facilities in question. Nevertheless, an investigation showing the impact of various weights is useful:
**Original objective equation**

```plaintext
minimize total_cost:
    # Costs to transport from center to plant
    (sum{k in CENTER, j in PLANT} a[k,j]*z[k,j]
    # Costs to center to transfer
    + sum{k in CENTER, i in TRANSFER} b[k,i]*x[k,i]
    # Costs to transport from transfer to plant
    + sum{i in TRANSFER, j in PLANT} c[i,j]*w[i,j]
    # Fixed costs associated with building each transfer facility
    + sum{i in TRANSFER} y[i]*F
    # Costs to process waste at transfer facility
    + sum{k in CENTER, i in TRANSFER} p*x[k,i]);
```

**New objective**

```plaintext
minimize total_cost:
    # Costs to transport from center to plant
    (sum{k in CENTER, j in PLANT} a[k,j]*z[k,j]
    # Costs to center to transfer
    + sum{k in CENTER, i in TRANSFER} b[k,i]*x[k,i]
    # Costs to transport from transfer to plant
    + sum{i in TRANSFER, j in PLANT} c[i,j]*w[i,j]
    # Fixed costs associated with building each transfer facility
    + sum{i in TRANSFER} y[i]*F
    # Costs to process waste at transfer facility
    + sum{k in CENTER, i in TRANSFER} p*x[k,i]);

# Weighting of environmental impact
    + P *( sum{j in PLANT} w[2,j] + sum{j in PLANT} w[1,j] + sum{j in PLANT} w[3,j]);
```

P is incrementally modified from 1 to 20, at which point it is noted that the cost no longer changes (see below).
From the above graph it can be easily seen that with a weight of 12, the cost is maximized at approximately $520,000. This can be qualitatively expressed as reducing environmental impact will not be more costly if it can be considered to be at least 12 times more important than minimizing total cost.

**Tradoffs and Real Life Applications**

The models defined here are a relatively simplistic approach to the issue of solid waste management and can appear to be fairly removed from the issue at hand. Indeed, as the author notes in the text, “Solid Waste Management”, ‘solid waste systems are highly individualistic” and are “highly interrelated with many other municipal systems, and decisions made in one have major impact on others”. Indeed, the concept of ‘cost’ in itself can take on different attributes throughout this model, including transportation costs, environmental impact (and cost), and, rather importantly, social and political cost. It is evident that reducing environmental impact is accompanied by higher costs, but who must shoulder the burden of cost is a difficult point to argue. Given also the point that waste processing facilities are often built in low income neighborhoods, environmental justice seems decidedly unfair.

What is reassuring, then, is noting the trends in waste generation have remained relatively unchanged in the past decade or so, at approximately 4lbs per individual (see
While the effort should be towards lessening the generation of waste, the increase in efforts towards recycling are also good news to bear. It is therefore important to proceed at these efforts with an enlightened view of what communities these efforts may serve better.

**Figure ES-1: MSW Generation Rates, 1960 to 2007**

![Graph showing MSW generation rates from 1960 to 2007](image)

**Figure 4.** Municipal solid waste generation rates, in a decade view. Municipal Solid Waste in The United States, 2007 Facts and Figures. Published by the United States Environmental Protection Agency
Code
The following AMPL code is from bottom half of the `recycle.mod` model file, which formulates the model. Note the first constraint is part of the new objective function that was made to assess the changes in cost due to reducing the amount of waste going through transfer facilities 1, 2 and 3.

```
minimize total_cost:
   sum{k in CENTER, j in PLANT} a[k,j]*z[k,j]
   + sum{k in CENTER, i in TRANSFER} b[k,i]*x[k,i]
   + sum{i in TRANSFER, j in PLANT} c[i,j]*w[i,j]
   + sum{i in INTOWN} 1*y[i]*F
   + sum{i in TRANSFER diff INTOWN} y[i]*F
   + sum{k in CENTER, i in TRANSFER} p*x[k,i]);

subject to limit: sum{j in PLANT} w[2,j] + sum{j in PLANT} w[1,j] +
   sum{j in PLANT} w[3,j] <= R;
subject to tcapacity{i in TRANSFER}: sum{k in CENTER} x[k,i] <=
   y[i]*Q;
subject to pcapacity{j in PLANT}: sum{k in CENTER} z[k,j] + sum{i in
   TRANSFER} w[i,j] <= G[j];
subject to mbalance{i in TRANSFER}: sum{k in CENTER} x[k,i] - sum{j
   in PLANT} w[i,j] = 0;
subject to useup{k in CENTER}: sum{i in TRANSFER} x[k,i] + sum{j in
   PLANT} z[k,j] = S;
```

The following is the RUN file used to run the model. Note the loop used to change the value of R (see above)

```
model recycle.mod;
data recycle.dat;
option solver cplex;
repeat {
solve;
display R > r.txt;
display total_cost > cost.txt;
display w,x,z;
display y;
display pollutant;
let R:=R-10;
} while R > 0;
```

The data file was not modified and therefore not reproduced here.